Algebra II – Simplifying Fractional Equations

Multiplying Binomials

F.O.I.L. – *F*irst *O*uter *I*nner *L*ast

Binomial Expansion

$(a+b)^n$ (n>0, n is an integer)

Coefficients for expansions of this form follow from the triangular array of numbers known as Pascal's Triangle. The numbers in the n^{th} row (beginning with row 0) form the coefficients of the expansion of $(a+b)^n$.

	Row
1	0
1 1	1
1 2 1	2
1 3 3 1	3
1 4 6 4 1	4
1 5 10 10 5 1	5

Numbers in the array are determined by the sum of the two numbers above and to the right and left. 1's make up the edges.

For example: $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$.

Note: $(a-b)^5 = (a+-b)^5 = a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$.

Multiplying Trinomials

$(a+b+c)(d+e+f) = ad + ae + af + bd + be + bf + cd + ce + cf$.

Factoring

$a^2 - b^2 = (a+b)(a-b)$.

Know your squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

Sums and Differences of Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$. $\quad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

Or, reworked, $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

Know your cubes: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

Order of Operations

Take care of powers, parentheses, multiplication and/or division, then addition and/or subtraction if the expression is written out from left to right. In the case of multiple operations which are of equal importance, work from left to right.

Products and Quotients of Rationals

$$\frac{a}{b}\cdot\frac{c}{d} = \frac{ac}{bd}. \qquad \frac{p}{q}\div\frac{r}{s} = \frac{p}{q}\cdot\frac{s}{r} = \frac{ps}{qr}. \qquad \frac{\frac{j}{k}}{\frac{l}{m}} = \frac{j}{k}\cdot\frac{m}{l} = \frac{jm}{kl}.$$

Powers

$$a^{-m} = \frac{1}{a^m}. \qquad b^m b^n = b^{m+n}. \qquad \frac{c^m}{c^n} = c^{m-n}. \qquad (d^m)^n = d^{mn}. \qquad e^{\frac{m}{n}} = \sqrt[n]{e^m}.$$

Canceling

$\frac{(x-y)a}{b(y-x)}$ is not in simplest form. Factoring out a –1 from either the numerator or denominator results in $\frac{(x-y)a}{-b(x-y)} = -\frac{a}{b}$.

In cases such as $\frac{\frac{ab}{cd}}{\frac{ae}{cf}}$, common factors like a can be canceled out automatically as well as common factors such as c. This will yield $\frac{\frac{b}{d}}{\frac{e}{f}}$ or ultimately $\frac{bf}{de}$.

Dividing Polynomials

Dividing polynomials is similar to dividing numbers. Example: $\frac{x^3 - 5x^2 + 4x - 2}{x-2}$ can be worked out using the long division algorithm as done at left:

$$\begin{array}{r} x^2 - 3x - 2 - \frac{6}{x-2} \\ x-2 \overline{\big) \; x^3 - 5x^2 + 4x - 2} \\ \underline{-x^3 + 2x^2} \\ -3x^2 + 4x \\ \underline{3x^2 - 6x} \\ -2x - 2 \\ \underline{+2x - 4} \\ -6 \end{array}$$

The dividend must always have all terms (even if the coefficient is zero) in decreasing degree of the variable.

Forms of Answers

All answers are to use the minimum number of grouping symbols ("( )", "[ ]", "{ }") possible. However, factored forms of an answer are acceptable if

1. the answer is completely factored.
2. each of the individual factors is simplified, and
3. the product of two or more monomials is expressed as a single monomial.

Acceptable	Not Acceptable
$4xy(1+3y+7xy^2)$	$(4x)(y)(1+3y-7xy^2)$
$4xy(-7xy^2+3y+1)$	$2^2xy(1+3y-7xy^2)$
$(x-3)^2(3x-1+3y+z)$	$(x-3)^2\{[(3x-1)+3y]+z\}$
$(x-3)(x-3)(3x-1+3y+z)$	