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Math 206 Midterm 1:1 page of notes (Chapters 7-8.5)

Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Average Value of f: (Mean Value Theorem) $\dfrac{1}{b-a}\int_a^b f(x)dx$

Average rate of change: $\dfrac{f(b)-f(a)}{b-a}$

Curve Sketching:

- Look for possible <u>extreme points</u> of *f(x)* by setting *f'(x)=0* and solving for *x*. Plug in that value into *f(x)* to obtain exact coordinates.
- Plug in the same value of *x* into *f''(x)* to obtain the <u>concavity</u>.
- Look for possible <u>inflection points</u> by setting *f''(x)=0* and solving for *x*. Plug in this value of *x* into *f(x)* to obtain exact coordinates.

Differentiating the Trig Functions:

$\dfrac{d}{dx}(\sin u) = \cos u\dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}(\cot u) = -\csc^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cos u) = -\sin u\dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}(\sec u) = \sec u\tan u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\tan u) = \sec^2 u\dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}(\csc u) = -\csc u\cot u\dfrac{du}{dx}$

Integrating the Trig Functions:

$\int \cos x dx = \sin x + c$ $\qquad \int \csc^2 x dx = -\cot x + c$

$\int \sin x dx = -\cos x + c$ $\qquad \int \sec x\tan x dx = \sec x + c$

$\int \sec^2 x dx = \tan x + c$ $\qquad \int \csc x\cot x dx = -\csc x + c$

$\int \cos^2 x dx = \dfrac{x}{2} + \dfrac{\sin 2x}{4} + c$ $\qquad \int \sin^2 x dx = \dfrac{x}{2} - \dfrac{\sin 2x}{4} + c$

The <u>Disk</u> Method

Volume of a solid of Revolution (about the x-axis): $V = \int_a^b \pi[f(x)]^2 dx$

Volume of a solid of Revolution (about the y-axis): $V = \int_c^d \pi[f(y)]^2 dy$

The <u>Washer</u> Method

Washer method about x-axis: $V = \int_a^b \pi([f(x)]^2 - [g(x)]^2)dx$

Washer method about y-axis: $V = \int_c^d \pi([F(y)]^2 - [G(y)]^2)dy$

The <u>Shell</u> Method

Shell method about x-axis: $V = \int_c^d 2\pi y[F(y) - G(y)]dy$

Shell method about y-axis: $V = \int_a^b 2\pi x[f(x) - g(x)]dx$

The Natural Logarithm Function: $\ln x = \int_1^x \dfrac{1}{t}dt$

Differentiation of the Natural Log: $\dfrac{d}{dx}[\ln g(x)] = \dfrac{1}{g(x)}\cdot\dfrac{d}{dx}g(x) = \dfrac{g'(x)}{g(x)}$

Integrating the Trig Functions using the Natural Logarithm Function:

$\int \tan u du = -\ln|\cos u| + c = \ln|\sec u| + c$

$\int \cot u du = \ln|\sin u| + c = -\ln|\csc u| + c$

$\int \sec u du = \ln|\sec u + \tan u| + c$

$\int \csc u du = -\ln|\csc u + \cot u| + c = \ln|\csc u - \cot u| + c$

Derivatives of Inverse Trig Functions:

$\dfrac{d}{dx}\sin^{-1} u = \dfrac{1}{\sqrt{1-u^2}}\cdot\dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}\cot^{-1} u = -\dfrac{1}{1+u^2}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}\cos^{-1} u = -\dfrac{1}{\sqrt{1-u^2}}\cdot\dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}\csc^{-1} u = -\dfrac{1}{|u|\sqrt{u^2-1}}\cdot\dfrac{du}{dx}$

$\dfrac{d}{dx}\tan^{-1} u = \dfrac{1}{1+u^2}\cdot\dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}\sec^{-1} u = \dfrac{1}{|u|\sqrt{u^2-1}}\cdot\dfrac{du}{dx}$

Integrals leading to Inverse Trig Functions: $\int \dfrac{du}{1+u^2} = \tan^{-1} u + c$ $\qquad \int \dfrac{du}{\sqrt{1-u^2}} = \sin^{-1} u + c$ $\qquad \int \dfrac{du}{u\sqrt{u^2-1}} = \sec^{-1}|u| + c$

The Exponential Function: $\log_e x = \ln x$ $\qquad$ If $y = \ln x$, then $e^y = x$

Integration by parts: $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

$\dfrac{d}{dx}e^u = e^u\dfrac{du}{dx}$ $\qquad \int e^{ax}du = \dfrac{1}{a}e^{ax} + c$ $\qquad \int e^u du = e^u + c$ $\qquad \int a^u du = \dfrac{a^u}{\ln a} + c$

Integrals Involving Trigonometric Powers:

$\int \sin^n u du = -\dfrac{\sin^{n-1} u\cos u}{n} + \dfrac{n-1}{n}\int \sin^{n-2} u du$

$\int \tan^n u du = \dfrac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u du$

$\int \sec^n u du = \dfrac{\sec^{n-1} u\sin u}{n-1} - \dfrac{n-2}{n-1}\int \sec^{n-2} u du$

$\int \cos^n u du = \dfrac{\cos^{n-1} u\cos u}{n} + \dfrac{n-1}{n}\int \cos^{n-2} u du$

$\int \cot^n u du = -\dfrac{\cot^{n-1} u}{n-1} - \int \cot^{n-2} u du$

$\int \csc^n u du = -\dfrac{\csc^{n-1} u\cos u}{n-1} - \dfrac{n-2}{n-1}\int \csc^{n-2} u du$

$$\int (\cos^m ax)(\sin^n ax)dx = \frac{\cos^{m-1} ax\sin^{n+1} ax}{(m+n)a} + \frac{m-1}{m+n}\int (\cos^{m-2} ax)(\sin^n ax)dx = -\frac{\sin^{n-1} ax\cos^{m+1} ax}{(m+n)a} + \frac{n-1}{m+n}\int (\cos^m ax)(\sin^{n-2} ax)dx$$

$$\int \frac{\cos^m ax}{\sin^n ax}dx = -\frac{\cos^{m+1} ax}{(n-1)a\sin^{n-1} ax} - \frac{m-n+2}{n-1}\int \frac{\cos^m ax}{\sin^{n-2} ax}dx = \frac{\cos^{m-1} ax}{a(m-n)\sin^{n-1} ax} + \frac{m-1}{m-n}\int \frac{\cos^{m-2} ax}{\sin^n ax}dx$$

$$\int \frac{\sin^m ax}{\cos^n ax}dx = \frac{\sin^{m+1} ax}{a(n-1)\cos^{n-1} ax} - \frac{m-n+2}{n-1}\int \frac{\sin^m ax}{\cos^{n-2} ax}dx = -\frac{\sin^{m-1} ax}{a(m-n)\cos^{n-1} ax} + \frac{m-1}{m-n}\int \frac{\sin^{m-2} ax}{\cos^n ax}dx$$

Basic Trigonometric Identities:

$\cos^2 x + \sin^2 x = 1$ $\qquad \sin^2 x = \dfrac{1-\cos 2x}{2}$ $\qquad \sin 2x = 2\sin x\cos x$

$\tan^2 x + 1 = \sec^2 x$ $\qquad \cos^2 x = \dfrac{1+\cos 2x}{2}$ $\qquad \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - \sin^2 x$

$\cot^2 x + 1 = \csc^2 x$

Robbins (1999)