Review of Trigonometry Concepts

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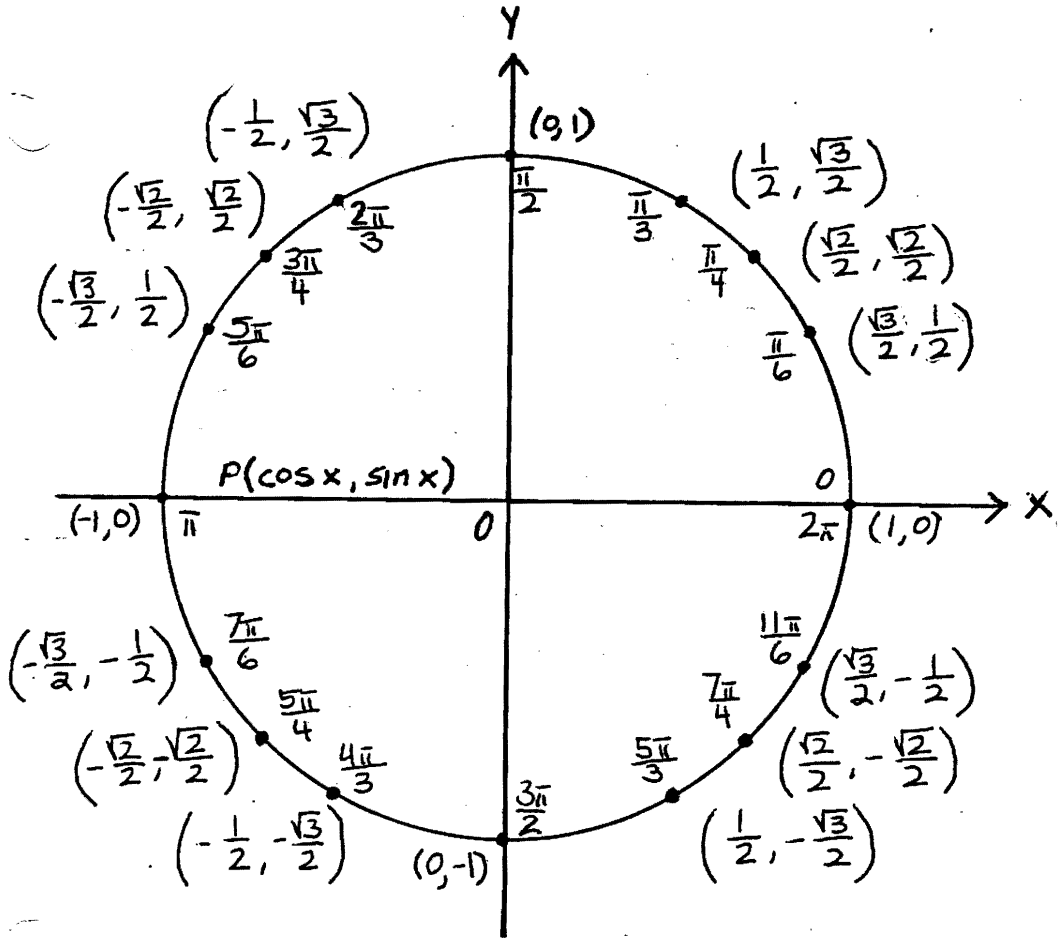
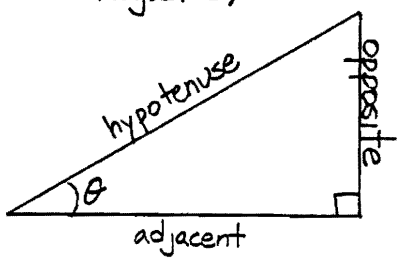




$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin\theta}$$

Pythagorean Identities:

$$\cos^2 x + \sin^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\cot^2 x + 1 = \csc^2 x$$

Sum/Difference of two angles:

$$\cos(x-y) = \cos x\cos y + \sin x \sin y$$
$$\cos(x+y) = \cos x\cos y - \sin x \sin y$$
$$\sin(x-y) = \sin x\cos y - \cos x \sin y$$
$$\sin(x+y) = \sin x\cos y + \cos x \sin y$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Double Angle:

$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$$

Reduction:

$$\sin(-x) = -\sin x \qquad \cot(-x) = -\cot x$$
$$\cos(-x) = \cos x \qquad \sec(-x) = \sec x$$
$$\tan(-x) = -\tan x \qquad \csc(-x) = -\csc x$$

Half Angle:

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$$

Product:

$$\sin x \sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$
$$\cos x \cos y = \tfrac{1}{2}[\cos(x-y) + \cos(x+y)]$$
$$\sin x \cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos x \sin y = \tfrac{1}{2}[\sin(x+y) - \sin(x-y)]$$

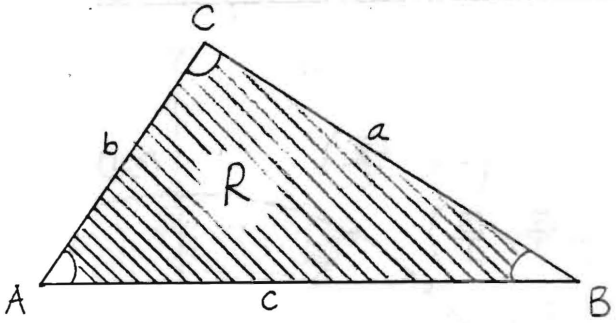
Area and sides of a triangle

Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



Area: $R = \frac{1}{2}ab\sin C$

$= \frac{1}{2}bc\sin A$

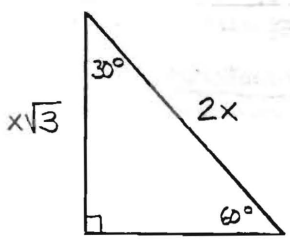
$= \frac{1}{2}ac\sin B$

Given 3 sides of a triangle & wish to find the area:

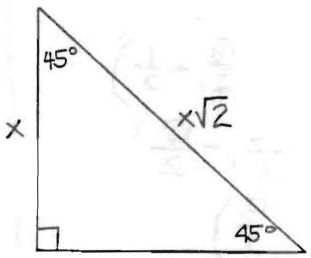
Heron's Theorem: $R = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{1}{2}$ the perimeter of the triangle $\left(s = \frac{a+b+c}{2}\right)$

Right Triangles: (Geometry Review)



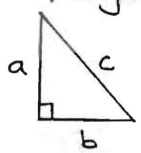
30-60 Right Triangle



Isosceles Right Triangle

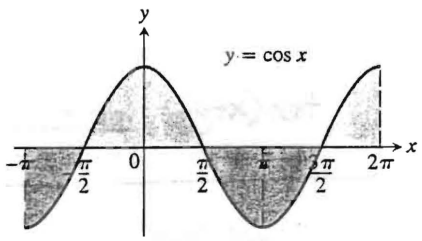
Area of an equilateral triangle: $\frac{x^2}{4}(\sqrt{3})$
where x = side of a triangle.

Pythagorean Theorem:



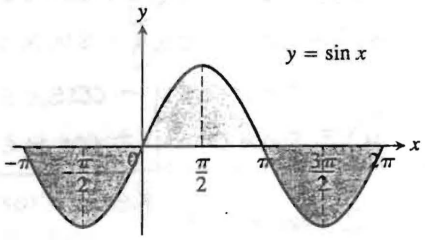
$a^2 + b^2 = c^2$

Principal functions and their graphs



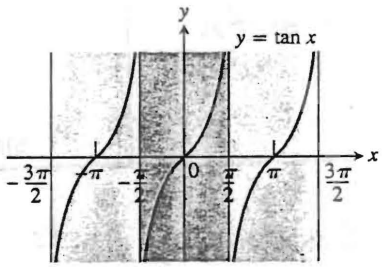
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π

(a)



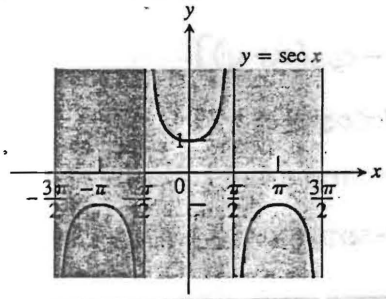
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π

(b)

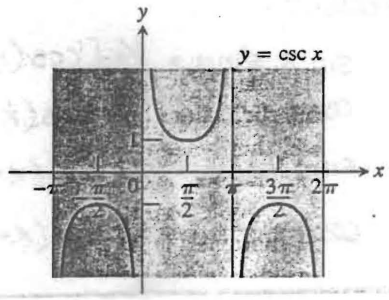


Domain: $x \ne \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $-\infty < y < \infty$
Period: π

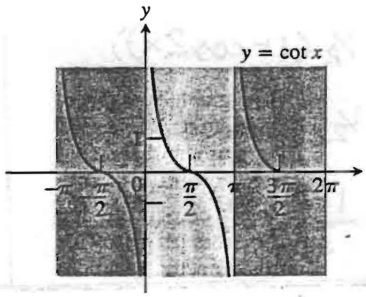
(c)



Domain: $x \ne \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $y \le -1$ and $y \ge 1$
Period: 2π



Domain: $x \ne 0, \pm\pi, \pm 2\pi, \ldots$
Range: $y \le -1$ and $y \ge 1$
Period: 2π



Domain: $x \ne 0, \pm\pi, \pm 2\pi, \ldots$
Range: $-\infty < y < \infty$
Period: π